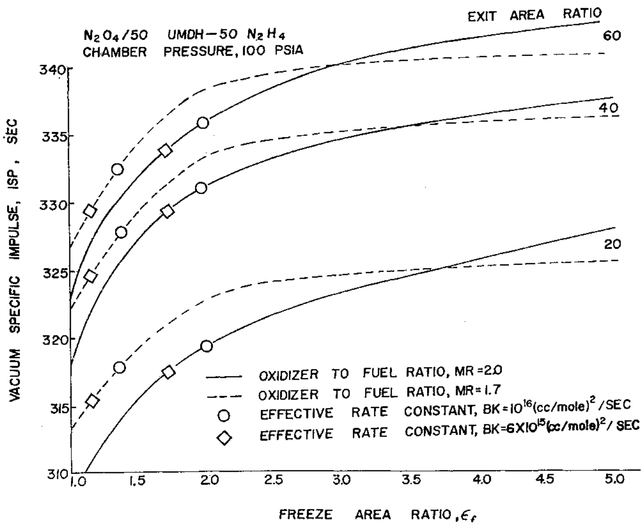

Fig. 2 Estimated theoretical vacuum specific impulse for a 20K thrust engine with chemical nonequilibrium.

Table 2 Specific impulse vs area ratio with kinetic losses

Exit area ratio	Mixture ratio, MR	I_{sp}, non-equilibrium	I_{sp}, theoretical shifting equilibrium
20	2.0	318	327
40	2.0	329	339
60	2.0	334	345
20	1.7	315	325
40	1.7	325	335
60	1.7	329	341

various freeze area ratios. These data are shown in Fig. 2. The estimated freeze area ratios from Fig. 1 also are shown for the rate coefficient 6×10^{15} $(cm^3/mole)^2/sec$ at several large exit area ratios. The effects of increasing the rate constant to 10^{16}, the upper limit on rate coefficient, also are shown.

The salient results are as follows:

1) In the region of the throat the rate of change of impulse with freeze area ratio is large and the lower mixture ratio theoretical performance is greater than the higher mixture ratio.

2) When kinetic considerations are superimposed on this graph, the performance at the higher mixture ratio is always greater than the lower, regardless of small perturbations in geometry or rate constant.

3) The performance loss is higher for larger exit area ratios and lower mixture ratios.

Results of Fig. 2 are summarized in Table 2. Other losses, such as combustion, drag, divergence, etc., can be added to the recombination losses by known methods to predict delivered I_{sp}. Thus, the kinetic scheme shown here provides a powerful tool for predicting the performance of space engines using the presently most attractive storable propellant combinations.

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Combined External and Internal Cooling

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Nomenclature

A_s	=	cooled surface area
c_{pc}	=	specific heat of coolant
d	=	wall thickness
h_c	=	coolant-side heat-transfer coefficient
h_c'	=	defined by Eq. (8)
h_g	=	gas-side heat-transfer coefficient without film cooling
h_g'	=	gas-side heat-transfer coefficient with film cooling
k	=	thermal conductivity of the wall
Q/A	=	heat flux
T_{ad}	=	adiabatic wall temperature without film cooling
T_{ad}'	=	adiabatic wall temperature with film cooling
T_{c_0}	=	temperature of the coolant in coolant manifold
T_c	=	temperature of coolant at exit of film-coolant injector
T_{w_c}	=	coolant-side wall temperature
T_{w_g}	=	gas-side wall temperature
$\dot{w}_c$	=	film-coolant flowrate
$\dot{w}_c^*$	=	film-coolant flowrate when $h_c = 0$
ϵ	=	dimensionless measure of distance from film-coolant injector, where wall temperature = T_c
η	=	defined by Eq. (9)
η^*	=	defined by Eq. (6)
ϕ	=	efficiency factor
ψ	=	efficiency factor

With the extensive use of hydrogen as a propellant in rocket engines, a potentially important method of cooling is gaseous film cooling, whereby hydrogen, injected through holes or slots in the chamber wall, is interposed between the wall and the primary gas stream. Film cooling usually is considered when heat transfer is so intense that external (convection) cooling, normally called regenerative cooling, alone is not satisfactory because of a high pressure drop of the coolant. Because some loss of rocket engine performance occurs with film cooling, a pertinent question is raised concerning the effectiveness of a combination of the two methods as compared with the use of film cooling alone.

Analysis

Eckert[1] states that the heat-transfer coefficient with film cooling, h_g', should be defined as

$$h_g' \equiv (Q/A)/(T_{ad}' - T_{w_g}) \qquad (1)$$

where T_{ad}' is the adiabatic wall temperature with film cooling. Using this definition, experiments made in Refs. 2–4 indicated that, under most conditions of practical interest, there is little difference in the heat-transfer coefficients with and without film cooling. Equation 1 then can be rewritten as

$$h_g \simeq (Q/A)/(T_{ad}' - T_{w_g}) \qquad (2)$$

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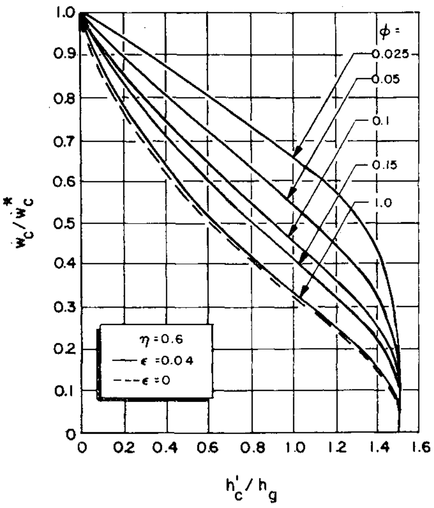

Fig. 1a $\dot{w}_c/\dot{w}_c^*$ **vs** h_c'/h_g **for** η **= 0.6 and various values of** ϕ**.**

The external convective heat-transfer coefficient is defined in the usual manner:

$$h_c \equiv (Q/A)/(T_{wc} - T_{c0}) \tag{3}$$

Eliminating Q/A in Eqs. (2) and (3) gives

$$h_g(T_{ad}' - T_{wg}) = h_c(T_{wc} - T_{c0}) \tag{4}$$

The calculation of T_{ad}' in Eq. (4) is a difficult problem, and many variables probably are involved. Although several correlations of T_{ad}' appear in the literature, none were obtained from experiments that matched rocket motor conditions. The closest simulations, to the author's knowledge, were the investigations done by NASA,[5, 6] where the data were correlated by

$$\phi(\ln\eta^* - \psi) = -[(h_gA_s/\dot{w}_c c_{pc}) - \epsilon] \tag{5}$$

In Eq. (5), ϕ and ψ are efficiency terms defined in Refs. 5 and 6 and are dependent upon many factors. In Eq. (5) ϵ represents a measurement of the distance downstream of the film-coolant injector, where the wall temperature at each point is equal approximately to the temperature of the coolant at the film-coolant injector. Normally $\phi \cong 0.1$ and, if the film coolant is injected parallel to the wall (which is optimum), $\psi = 0$. In Eq. (5)

$$\eta^* \equiv (T_{ad} - T_{ad}')/(T_{ad} - T_c) \tag{6}$$

where T_{ad} is the normal adiabatic wall temperature. Eliminating T_{ad}' in Eqs. (4) and (5), an expression for $\dot{w}_c$ is obtained. It is

$$\dot{w}_c = \frac{-(A_sh_g/c_{pc})}{\phi[\ln\{\eta - [(T_{wg} - T_{c0})/(T_{ad} - T_c)](h_c'/h_g)\} - \psi] - \epsilon} \tag{7}$$

where

$$h_c' \equiv 1/[(1/h_c) + (d/k)] \tag{8}$$

$$\eta \equiv (T_{ad} - T_{wg})/(T_{ad} - T_c) \tag{9}$$

In most applications

$$T_{wg} - T_{c0} \simeq T_{wg} - T_c$$

Thus, Eq. (7) may be rewritten

$$\dot{w}_c = \frac{-(A_sh_g/c_{pc})}{\phi\{\ln[\eta - (1 - \eta)(h_c'/h_g)] - \psi\} - \epsilon} \tag{10}$$

When $h_c = 0$, Eq. (10) reduces to

$$\dot{w}_c^* = \frac{-(A_sh_g/c_{pc})}{\phi^*(\ln\eta^* - \psi) - \epsilon} \tag{11}$$

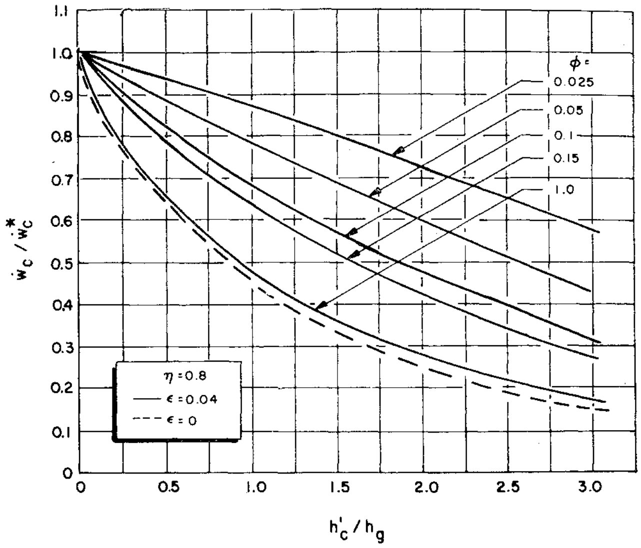

Fig. 1b $\dot{w}_c/\dot{w}_c^*$ **vs** h_c'/h_g **for** η **= 0.8 and various values of** ϕ**.**

which, with $\epsilon = 0.04$, is the NASA result.[6] The * indicates the value of the parameter when $h_c = 0$.

In order to determine conveniently the effect of the external coolant for the special case of tangential injection through a slot ($\psi = 0$), and, for $\epsilon = 0.04$, Eq. (10) was divided by Eq. (11). Thus,

$$\frac{\dot{w}_c}{\dot{w}_c^*} = \frac{\phi^* \ln\eta - 0.04}{\phi \ln[\eta - (h_c'/h_g)(1 - \eta)] - 0.04} \tag{12}$$

Note that in Eq. (12), so that $\dot{w}_c$ can be compared with $\dot{w}_c^*$, $T_{ad}' = T_{wg}$ and, consequently, $\eta^* = \eta$. That is to say, the comparison is made for the same gas-side wall temperatures.

Results

In order to evaluate the effectiveness of a combination of internal and external cooling relative to the use of film-cooling alone, Figs. 1a and 1b, based on Eq. (12), were prepared. The parameter η is a constant value in each of the figures, and the dimensionless film-coolant flowrate $\dot{w}_c/\dot{w}_c^*$ is plotted for several values of ϕ vs the coefficient h_c', divided by the gas-side, heat-transfer coefficient, h_g. For simplicity and without the introduction of a large error in Fig. 1, the assumption was made that $\phi^* = \phi$. Curves also are presented for $\epsilon = 0$, as there is some evidence from rocket engine film-cooling experiments that a value of $\epsilon = 0.04$ may be too large. The x-axis intercept, a point common to all curves in Fig. 1, corresponds to 100% regenerative cooling. For a given value of h_c'/h_g, regenerative cooling offers a greater reduction of the required film-coolant flowrate when η is low (Fig. 2).

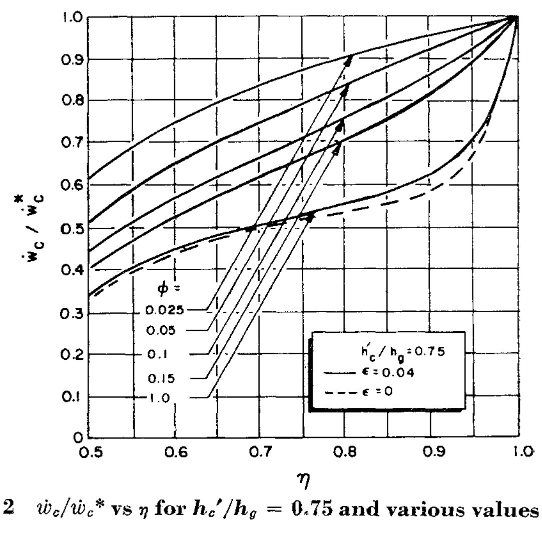

Fig. 2 $\dot{w}_c/\dot{w}_c^*$ **vs** η **for** h_c'/h_g **= 0.75 and various values of** ϕ**.**

As an illustration of the results of the calculations, assume η lies between 0.6 and 0.8 and $h_c'/h_g = 1.0$. With these data and referring to Fig. 1, it can be seen that the reduction in the film-coolant flowrate, as a result of the combined use of regenerative and gaseous film cooling, will be between 30 and 55%. These results are based upon $\phi = 0.1$, which experiments have indicated to be a typical value. Because a completely film-cooled chamber is of relatively simple construction and consequently is potentially capable of high reliability, the previously mentioned reduction of film-coolant flow must be weighted against the increase in complexity if the regenerative cooling system is added.

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Gaseous Film Cooling with Multiple Injection Stations

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Nomenclature

A_s = cooled surface area
c_{pc} = specific heat of coolant
d = wall thickness
h_c = coolant-side heat transfer coefficient
h_c' = $1/(1/h_c + d/k)$
h_g = gas-side heat transfer coefficient without film cooling
k = thermal conductivity of the wall
n = number of slots
s = distance downstream of last slot
T_{ad} = adiabatic wall temperature without film cooling
T_{ad}' = adiabatic wall temperature with film cooling
T_c = temperature of coolant at exit of film-coolant injector
T_{wg} = gas-side wall temperature
V_c = velocity of coolant at exit of film-coolant injector
V_g = velocity of gas at the point where the film coolant is introduced
$\dot{w}_c$ = coolant flow rate
ϵ = dimensionless measure of distance from film coolant injector where wall temperature equals T_c
η = defined by Eq. (2a)
η^* = defined by Eq. (1a)
ϕ = efficiency factor
ψ = efficiency factor

When wall temperatures are lowered by using gaseous film cooling, it is usually necessary or wise to inject the coolant along the wall through a series of openings in the wall.

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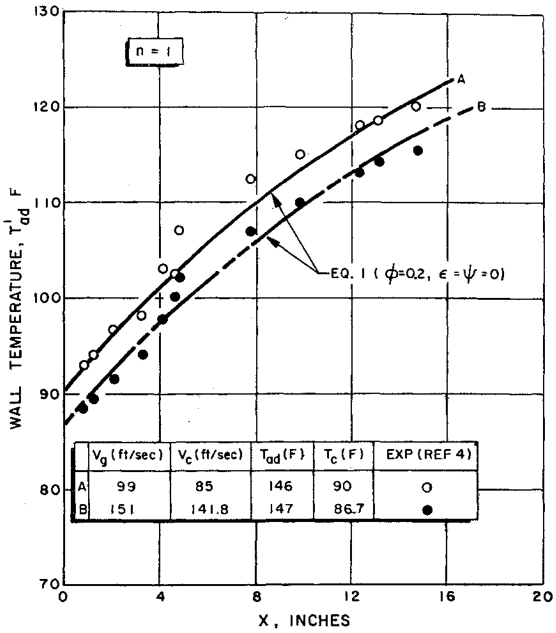

Fig. 1 Wall temperature vs distance downstream of slot, $n = 1$.

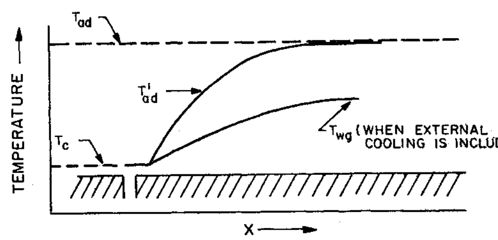

Fig. 2 Adiabatic wall temperature, $n = 1$.

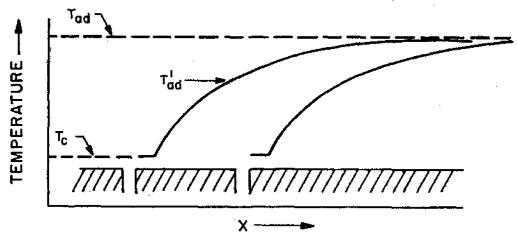

Fig. 3 Adiabatic wall temperatures, $n = 2$.

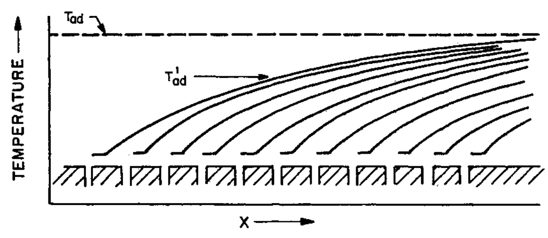

Fig. 4 Adiabatic wall temperature, $n = 11$.

In the establishment of an analytical model for the multiple injection case, one would expect that it is somewhat more complicated than when the coolant is injected at only one station. The coolant introduced at a particular station will not only affect the temperature of the wall immediately downstream of its injection point, but its influence will extend by a progressively lessening degree beyond successive film-coolant injection stations.

The thermal protection achieved downstream from coolant injected at a single injection station has been determined experimentally for a wide range of coolant-gas conditions, but